than Number; unless we accept the Pythagorean notion—that "*Number* is the essence of *things*."

Most definitions and illustrations of *Concrete Numbers* are ambiguous. Prof. Simon Newcomb (see Alg. p. 3) says, "A concrete number is one in which the kind of *quantity* which it measures" (?) "is expressed or understood; as, Seven *miles*, Three *days*." (*Italics his.*)

Webster says, "Concrete number is a number *associated with*, or *applied to* a particular object; as, *Three* men, *five* days." Which of the two is correct? Do the italicised words of the former or of the latter express *numbers?* If the latter, then the so-called *Denominate Numbers* of our Arithmetics should be "Denominate Quantities."

Now, if we define division as a process of finding one of the equal parts of a *number*, may we not substitute *concrete quantity* for *number?* A fourth part of \$20 is \$5 and four times \$5 are \$20.

We wish to know what is a fourth part of 5768 acres. How shall we find it? What name shall we give to the process? Prof. Ellwood would say "Take a fourth part of the *number* 5768 and annex the name of the concrete unit.

We suggest that a *concrete number* cannot be used as a multiplier, because it is a *quantity* and *not* a *number*.

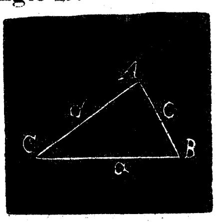
OBLIQUE ANGLED TRIANGLES.

By RALPH H. KUNSTADTER, Graduate Student, Yale University, New Haven, Connecticut.

Solution of an oblique angled triangle, given the two sides and the angle included between them.

The formula found below, was not known to me being in use, and have ascertained its existence only after having derived it originally in a simple way from the fundamental theorem of trigonometry, namely; the proportion of the sines of the angles is direct with that of the sides opposite to them.

Given in an oblique angled triangle for instance, the sides a, b, and angle C, and we wish to find directly from the given quantities the angle B.



$\frac{a}{b}=\frac{\sin A}{\sin B}\ldots.(1)$. Equation (1) has two unknown quantities, we must therefore get rid of A, in order to solve our example. $\frac{a}{b}=\frac{\sin[180^\circ-(B+C)]}{\sin B}=\frac{\sin(B+C)}{\sin B}\ldots(2)$

$\frac{a}{b}=\frac{\sin B\cos C+\cos B\sin C}{\sin B}\ldots.(3)=\cos C+\cot B\sin C$

$\ldots(4)$. $\cot B=\frac{a}{b\sin C}-\cot C\ldots.(5)$. In order to make equation (5) convenient for the application of logarithmic tables, assume $\frac{a}{b\sin C}=\cot\phi$. Hence,

we have $\cot B = \cot\phi - \cot C \ldots (6)$ and also $\cot B = -\dfrac{\sin(\phi - C)}{\sin\phi \sin C} \ldots (7)$ or $\tan B = \dfrac{\sin\phi \sin C}{\sin(C-\phi)} \ldots (8)$.

As to the practical value of this formula, the less we have to open the book of logarithms the better the formula is, and therefore $\tan\dfrac{B-A}{2} = \dfrac{b-a}{b+a}\cot\dfrac{C}{2}$ is preferable to the above, but as regards the theoretical rank, it is perhaps of the same degree.

It is interesting to notice that an important formula can also be obtained with the same processes from the oblique-angled spherical triangle, (but only for limited cases). Given again a, b, C, by drawing a spherical triangle and lettering it the same as in the above figure,

$\dfrac{\sin a}{\sin b} = \dfrac{\sin A}{\sin B} \ldots (\mathrm{I}) = \dfrac{\sin(B+C)}{\sin B} \ldots (\mathrm{II})$. $\dfrac{\sin a}{\sin b} = \cos C + \cot B \sin C \ldots$ (III), $\cot B = \dfrac{\sin a - \sin b \cos C}{\sin b \sin C} = \dfrac{\sin a}{\sin b \sin C} - \cot C$.

We might again assume $\dfrac{\sin a}{\sin b \sin C} = \cot\phi$, and proceed as above, or find the tangent or cotangent of the sum of the two unknown angles divided by two.

NON-EUCLIDEAN GEOMETRY, HISTORICAL AND EXPOSITORY.

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CHAPTER FIRST.

EUCLID.

When Alexander the Great was conquering the world, from Macedonia to the Indus, from the Caspian sea to the cataracts of the Nile, founding at least eighteen cities named Alexandria, little did he think that with the only one of these the name now suggests would be connected a man destined to give his name to the universe; for all spaces are now Euclidean or Non-Euclidean.

Euclid, after the death of Alexander, was called by Ptolemy Lagus to open the mathematical school of the first true university, that at Alexandria, and on its teaching so impressed his individuality, that henceforth his name and his immortal Elements stood for his science itself.

Says De Morgan: "As to writing another work on geometry, the middle ages would as soon have thought of composing another New Testament. ....his order of demonstration was thought to be necessary, and founded in the nature of our minds.